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# A Comparison of Two Models of the $\rho$-Meson†

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*Abstract*

A renormalization of the $\rho$-propagator is presented. It is shown that if the $p$-wave, isovector $\pi$–$\pi$ amplitude is assumed to be dominated by this renormalized $\rho$, many scattering parameters are predicted that agree well with experimental data. The model is compared with one presented by Tschang and Parkinson. It is shown that the predictions of the two models are the same, but that the renormalization model does not contain some of the theoretical problems of the Tschang and Parkinson scheme.

## 1. *Introduction*

In 1967, Ball and Parkinson (Ball and Parkinson, 1967) introduced a parametrized model of the $\rho$-meson. The model treated the $\pi$-$\pi$ amplitude as a sum of contributions from several two-particle channels. However, this model required the use of a numerical cut-off to prevent various integrals from diverging, and the formalism could not handle scattering channels of total angular moment greater than one.

In 1971, Tschang and Parkinson (hereafter referred to as TP) (Tschang and Parkinson, 1971) extended the original model, eliminating the restrictions described above. They obtained a parametrized model of the $\rho$ by fitting their model to low energy $\pi$-$\pi$ phase shift data (Baton, Laurens, and Reignier, 1970).

In this paper, I present a model of the $\rho$-meson propagator obtained by a straightforward renormalization technique. The predictions of this renormalization model are compared with those of the TP model and experimental data.

In Section 2, the TP model is considered. Section 3 deals with the renormalization model. The predictions of the $p$-wave, isovector phase shifts of the two schemes are discussed in these sections. Section 4 is a brief discussion of the predictions of the pion form factor and $p$-wave scattering length of these models.

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## 2. *The Tschang-Parkinson Model*

Using the $K$-matrix formalism, TP have parametrized the $\pi$-$\pi$ scattering amplitude in the $I = 1$, isovector channel as

$$T_{\pi\pi}(s) = \frac{g^2}{s - m^2 + \sum_{j=1}^{5} C_j^2 V_j(s) + i \sum_{j=1}^{5} C_j^2 \rho_j(s)\Theta(s - t_j)} \tag{2.1}$$

where $m$ is the resonance mass ($m_\rho$) and the function $\rho_j(s)$ is the phase space function for the $j$th channel contribution to $\pi$-$\pi$ scattering. $V_j(s)$ is the real part of a dispersion integral for the $j$th channel, and is given in the appendix of TP. The number $t_j$ is the threshold energy in the appropriate channel, and in terms of the particle masses, is

$$t_j = [m_j(1) + m_j(2)]^2 \tag{2.2}$$

Also occurring in the phase space function is the threshold

$$u_j = [m_j(1) - m_j(2)]^2 \tag{2.3}$$

The coefficients $C_j^2$ were adjusted to give the best fit to the low energy $\pi$-$\pi$ phase shift data. Table 1 is a reproduction of the table in TP giving the appropriate quantities in equation (2.1). In Table 1 and throughout this paper, $\mu$ will be used as the pion mass.

The phase shifts are defined by

$$\tan \delta(s) = -\mathrm{Im}\, D(s)/\mathrm{Re}\, D(s) \tag{2.4}$$

where $D(s)$ is the denominator of equation (2.1).

As can be seen from the threshold values in Table 1, only the $\pi$-$\pi$ channel contributes to the imaginary part of $D(s)$ for almost the entire range of energy considered. The real part of $D(s)$ is not particularly sensitive to the contributions from the various channels at low energies. Thus, the $\pi$-$\pi$ channel should

TABLE 1. Values of parameters and phase space functions required by TP model to fit phase shift data

| Channel number | Particles | $t_j/\mu^2$ | $u_j/\mu^2$ | $C_j^2$ | $\rho_j(s)$ |
|---|---|---|---|---|---|
| 1 | $\pi\pi$ | 4 | 0 | .99 | $(s - t_1)^{3/2}(16s)^{-1/2}$ |
| 2 | $\pi\omega$ | 43.7 | 21.2 | .19 | |
| 3 | $\rho\eta$ | 88.4 | 2.37 | .063 | $\frac{[(s - t_j)(s - u_j)]^{3/2}}{16s}$ |
| 4 | $K^*\bar{K}$ | 98.8 | 7.9 | .19 | |
| 5 | $N\bar{N}(+-)^*$ | 180.6 | 0 | 7.72 | $[s(s - t_5)/4]^{1/2}$ |

* (+−) are the helicities of the nucleons.

be sufficient to describe the phase shift data almost completely. Since the threshold for the $\pi$-$\omega$ channel is 930 MeV ($t_2/\mu^2 = 43.7$) the $\pi$-$\omega$ channel should affect the phase shift predictions of the TP model only above 930 MeV. None of the other channels should have any effect. A simple numerical investigation confirms this.

The phase shift predictions of equation (2.1) were tested to see what effect various changes in $D(s)$ would have on these predictions. As expected, I found

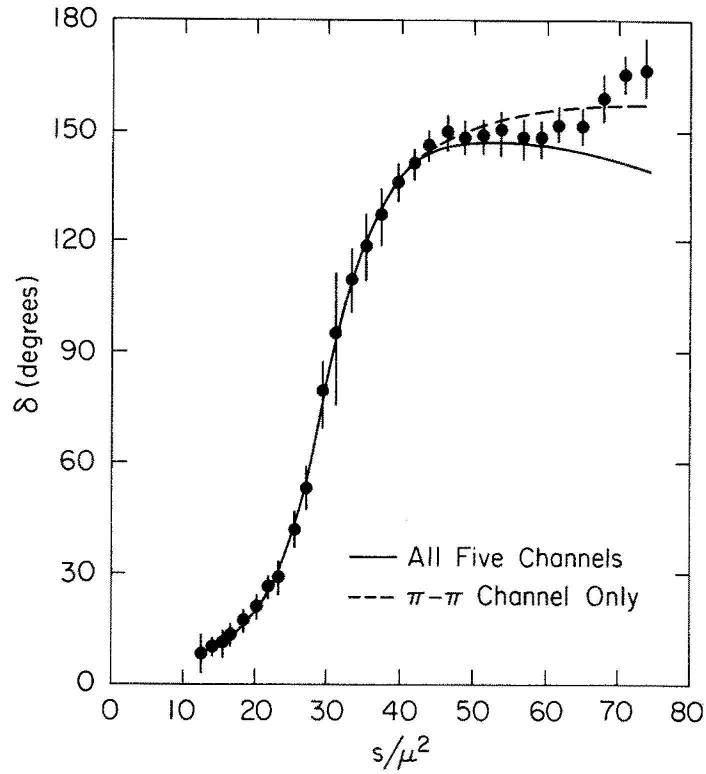



Figure 1—Predictions of the $\pi$-$\pi$ isovector phase shifts for the full 5-channel TP model, single $\pi$-$\pi$ channel TP model, and experimental data.

that if all but the $\pi$-$\pi$ channel are omitted, the phase shift values do not change much from the predictions when all five channels are included. As would be expected, the only noticable change occurs at higher energies. This is illustrated in Figure 1, where the phase shift curve is plotted for the case when all five channels are included in $D(s)$, and when the denominator contains only the $\pi$-$\pi$ channel contributions. When the $\pi$-$\omega$ channel is included with the $\pi$-$\pi$ channel, the phase shift predictions are identical to the five channel case. It is found that omitting or including any of the other three channels ($\rho\eta$, $K^*\bar{K}$, $N\bar{N}$) makes no change in any of the two channel predictions. Thus, the $\pi$-$\pi$

and $\pi$-$\omega$ channels are the entire contributors to the phase shift predictions of the TP model up to energies ~1100 MeV.

It had been pointed out by Ball and Parkinson (Ball and Parkinson, 1967) that the $N\bar{N}$ channel is the most important contribution to the creation of the $\rho$. Since omission or inclusion of this channel has no effect on the $\pi$-$\pi$ data, such a conclusion seems to be incorrect.

I have also looked at the denominator of equation (2.1) for negative $s$. I find a pole in $T_{\pi\pi}$ at $s = -107.66\mu^2$. The left hand cut in the TP model has been approximated by a pair of complex conjugate poles. If the pole positions are defined to be at $s = x \pm iy$, TP find that $-5\mu^2 < x < 0$ and $20\mu^2 < y < 25\mu^2$. Thus, the pole at $s = -107.66\mu^2$ does not come from the left hand cut. Further investigation as to its origin is in order.

## 3. *Renormalization Model*

In this Section, I describe a renormalization technique for the $\rho$ propagator. Only the two pion contribution is considered, so the results will be compared to the TP predictions containing just the $\pi$-$\pi$ channel.

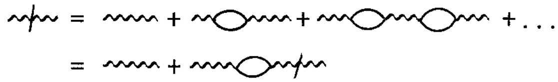
The renormalized $\rho$ propagator is taken to be a bare propagator adjusted by a sum of two-pion bubbles. The renormalized propagator will be represented diagramatically by

$D_{\mu\nu}(s) =$  (3.1a)

and the bare propagator by

$D^0_{\mu\nu}(s) =$  (3.1b)

The equation for the renormalized propagator, to lowest order in the $\rho\pi\pi$ coupling constant, is (Taylor, 1963; Buchl and Nigam, 1972)

 (3.2)

I am approximating the $\rho\pi\pi$ vertex by a constant, and assuming that for low energies, it is sufficient to keep only those terms which are to lowest order in this coupling constant.

Analytically,

$$D^0_{\mu\nu}(s) \equiv [g_{\mu\nu} - p_\mu p_\nu/s](s - m_0{}^2 + i\epsilon)^{-1} \tag{3.3}$$

where $s = p^2$ is the off-shell 4-momentum squared, and $m_0$ is the bare $\rho$ mass. Since $s \neq m_0{}^2$ off the mass shell, the propagator in equation (3.3) is a pure $p$-wave propagator. This form is necessary to yield a pure $p$-wave $\rho$ when renormalized. To see that the bare propagator in equation (3.3) is pure $p$-wave, consider the inner product.

$$q_\mu D^0_{\mu\nu}(s) q'_\nu \sim q_\mu (g_{\mu\nu} - p_\mu p_\nu/s) q'_\nu \tag{3.4}$$

In the "rest frame" of the off-shell $\rho$, $\rho_\mu = (p_0, \mathbf{0})$. Thus

$$q_\mu D^0_{\mu\nu} q'_\nu \sim -\mathbf{q} \cdot \mathbf{q}' \sim \cos\theta$$

If the tensor term in equation (3.3) were $p_\mu p_\nu / m_0{}^2$, $q_\mu D^0_{\mu\nu} q'_\nu$ would contain a term which was independent of $\cos\theta$ in addition to a $\cos\theta$ term. This would represent a particle with mixed $s$ and $p$ wave components.

In equations (3.1a) and (3.2), let $D_{\mu\nu} \equiv A g_{\mu\nu} + B p_\mu p_\nu$, and define the two-pion bubble as

 $$\equiv \Pi_{\mu\nu} \equiv I_1 g_{\mu\nu} + I_2 p_\mu p_\nu$$

Then equation (3.2) becomes

$$D_{\mu\nu} = D^0_{\mu\nu} + D^0_{\mu\lambda} \Pi_{\lambda\sigma} D_{\sigma\nu} \tag{3.5}$$

This yields

$$A = (s - m_0{}^2 - I_1(s))^{-1} \tag{3.6a}$$

and

$$B = -[s(s - m_0{}^2 - I_1(s))]^{-1} \tag{3.6b}$$

Using the bare Lagrangian

$$L = g_0 \rho_\mu \cdot (\phi x\, \partial_\mu \phi)$$

one finds

$$\Pi_{\mu\nu} = I_1 g_{\mu\nu} + I_2 p_\mu p_\nu = \\ i g_0{}^2 (2\pi)^{-4} \int d^4k (p - 2k)_\mu (p - 2k)_\nu (k^2 - \mu^2)^{-1} [(p-k)^2 - \mu^2]^{-1} \tag{3.6c}$$

$\mu$ and $\nu$ being the Lorentz indices associated with the incoming and outgoing $\rho$ fields. There is a factor 2 from the isospin contribution to the two-pion bubble which is cancelled by 1/2! a symmetry factor.

To evaluate $\Pi_{\mu\nu}$, note that the projection operator

$$[g_{\mu\nu} - p_\mu p_\nu / s]/3$$

projects out the $g_{\mu\nu}$ part of any two index quantity. Thus

$$I_1 = \tfrac{1}{3} [g_{\mu\nu} - p_\mu p_\nu / s]\, \Pi_{\mu\nu}$$

Similarly

$$I_2 = -\frac{1}{3s} [g_{\mu\nu} - 4 p_\mu p_\nu / s]\, \Pi_{\mu\nu}$$

In addition to these projection operators, the following identities are also useful (see, for example, Schweber, 1961)

$$[(p-k)^2-\mu^2]^{-1} = (k^2-\mu^2)^{-1} - (p^2-2p.k)(k^2-\mu^2)^{-1}[(p-k)^2-\mu^2]^{-1} \tag{3.7a}$$

$$\int d^4k k_\mu k_\nu f(k^2) = \tfrac{1}{4}g_{\mu\nu}\int d^4k k^2 f(k^2) \tag{3.7b}$$

$$\int d^4k k_\mu k_\nu k_\lambda k_\sigma f(k^2) = \tfrac{1}{24}[g_{\mu\nu}g_{\lambda\sigma}+g_{\mu\lambda}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\lambda}]\int d^4k(k^2)^2 f(k^2) \tag{3.7c}$$

and

$$\int d^4k\,(\text{odd number of } k \text{ 4-vectors})\,f(k^2) = 0 \tag{3.7d}$$

Defining the quadratically, and logarithmically divergent quantities

$$Q_0 \equiv \int d^4k(k^2-\mu^2+i\epsilon)^{-1} \qquad L_0 \equiv \int d^4k(k^2-\mu^2+i\epsilon)^{-1} \tag{3.8}$$

I obtain

$$\begin{aligned} I_1 = ig_0{}^2(2\pi)^{-4}[Q_0 + (\mu^2 - s/3)L_0 - i\pi^2 s(1/6 + s/(3\mu^2)) \\ -\tfrac{4}{3}\int d^4k(s-2p.k)^3\{k^2-(p.k)^2/s\}(k^2-\mu^2+i\epsilon)^{-4}\{(p-k)^2- \\ -\mu^2+i\epsilon\}^{-1}] \equiv ig_0{}^2(2\pi)^{-4}[Q_0+(\mu^2-s/3)L_0+F_1(s)] \end{aligned} \tag{3.9}$$

and

$$I_2 = ig_0{}^2(2\pi)^{-4}[L_0/3 + F_2(s)]$$

The functions $F_1(s)$ and $F_2(s)$ are finite. As can be seen from equations (3.6a) and (3.6b), $I_2$, and therefore $F_2$ does not contribute to the renormalized propagator. It is, therefore, unnecessary to discuss $F_2$.

The integral in $F_1(s)$ can be evaluated using the Feynman identity

$$(ab^n)^{-1} = n\int_0^1 dx(1-x)^n[ax+b(1-x)]^{-n-1}$$

Referring to equations (3.6a) and (3.6b), the mass renormalization is achieved by setting

$$m_0{}^2 + I_1(s) = m^2 + \delta m^2 + [I_1(s) - I_1(m^2)] + \operatorname{Re} I_1(m^2) + i \operatorname{Im} I_1(m^2)$$

and letting $\delta m^2 + \operatorname{Re} I_1(m^2) = 0$. Then

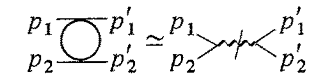
$$m_0{}^2 + I_1(s) = m^2 + \operatorname{Re}[I_1(s) - I_1(m^2)] + i \operatorname{Im} I_1(s) \tag{3.10}$$

By making a Wick rotation, it is easily seen that $Q_0$ and $L_0$ are purely imaginary. Thus, from equation (3.9)

$$\operatorname{Im} I_1(s) = g_0{}^2(2\pi)^{-4} \operatorname{Re}[Q_0 + (\mu^2 - s/3)L_0 + F_1] = g_0{}^2(2\pi)^{-4} \operatorname{Re} F_1(s) \tag{3.11}$$

It is also noted that Re $[I_1(s) - I_1(m^2)]$ does not contain the quadratically divergent term. That is, mass renormalization removes the quadratic divergences.

To remove the logarithmic divergences, the coupling constant is renormalized by multiplication by a renormalization constant $Z$. The two-pion amplitude is assumed to be dominated by the $\rho$ in the energy region under investigation. This is written diagramatically as



(3.12)

which represents the expression

$$T_{\pi\pi} \simeq g_0{}^2(p_1 - p_2)_\mu D_{\mu\nu}[(p_1 + p_2)^2](p_1' - p_2')_\nu \tag{3.13}$$

The coupling constant is renormalized by writing

$$T_{\pi\pi} \simeq (Zg_0{}^2)(p_1 - p_2)_\mu Z^{-1} D_{\mu\nu}[(p_1 + p_2)^2](p_1' - p_2')_\nu \tag{3.14}$$

and defining the renormalized coupling constant by

$$g^2 = Zg_0{}^2 \tag{3.15}$$

Referring to equations (3.6a), (3.6b), and (3.10), $Z^{-1}D_{\mu\nu}$ will contain the factor

$$Z^{-1}[s - m^2 - \operatorname{Re}\Delta I_1 - i \operatorname{Im} I_1(s)]^{-1} \tag{3.16}$$

where $\Delta I_1 = I_1(s) - I_1(m^2)$.

A simple pole at the $\rho$ mass is insured by requiring

$$\lim_{s\to m^2} Z[s - m^2 - \operatorname{Re}\Delta I_1] \to s - m^2$$

or

$$Z = [1 - \operatorname{Re} dI_1/ds|_{s=m^2}]^{-1} \tag{3.17}$$

Define a function $\Lambda(s)$ so that

$$Z[s - m^2 - \operatorname{Re}\Delta I_1 - i \operatorname{Im} I_1(s)] = s - m^2 + \Lambda(s) \tag{3.18}$$

Then, using equation (3.17),

$$\Lambda(s) = Z[(s - m^2) \operatorname{Re} dI_1/ds|_{m^2} - \operatorname{Re}\Delta I_1 - i \operatorname{Im} I_1(s)] \tag{3.19}$$

As seen in equation (3.9), $I_1$ contains the overall multiplicative factor $g_0{}^2$. Thus, every term in $\Lambda(s)$ contains the factor $(Zg_0{}^2) = g^2$. Therefore,

$$\Lambda(s) = [(s - m^2) \operatorname{Re} dI_1/ds|_{m^2} - \operatorname{Re}\Delta I_1 - i \operatorname{Im} I_1(s)]_{g_0{}^2 \to g^2} \tag{3.20}$$

From equations (3.9) and (3.11), I obtain

$$\operatorname{Re}\Lambda(s) = -(s - m^2)\operatorname{Im} g^2(2\pi)^{-4}[F_1'(m^2) - (F_1(s) - F_1(m^2))/(s - m^2)] \tag{3.21a}$$

and

$$\operatorname{Im}\Lambda(s) = -g^2(2\pi)^{-4}\operatorname{Re} F_1(s) \tag{3.21b}$$

Thus, $\Lambda$ is finite. Therefore, so are the functions $A$ and $B$ of equations (3.6a) and (3.6b). The renormalized propagator is

$$D_{\mu\nu}(s) = [g_{\mu\nu} - p_\mu p_\nu/s](s - m^2 + \Lambda(s))^{-1} \tag{3.22}$$

Using this to describe the approximate isovector, $p$-wave amplitude, the phase shifts are found from

$$\tan\delta = -\operatorname{Im}\Lambda(s)[s - m^2 + \operatorname{Re}\Lambda(s)]^{-1} \tag{3.23}$$

An evaluation of $F_1(s)$ yields

$$\operatorname{Im}\Lambda(s) = g^2(48\pi)^{-1}(s - 4\mu^2)^{3/2}(s)^{-1/2}\Theta(s - 4\mu^2) \tag{3.24a}$$

and

$$\operatorname{Re}\Lambda(s) = g^2(3\pi^2)^{-1}(s - m^2)\{(1/4 + \mu^2/m^2)q_m{}^2 W_R(m^2) + q_m{}^2 W_R'(m^2)\mu^2/m^2 - [sq_s{}^4 W_R(s) - m^2 q_m{}^4 W_R(m^2)]/(s - m^2)\} \tag{3.24b}$$

where $q_s^2 = (s - 4\mu^2)/(4s)$, $q_m{}^2 = q_{s=m^2}^2$, $W_R'(s) = dW_R/dq_s{}^2$, and

$$W_R(s) = P\int_{-1}^{1} dy(y^2 - 4q_s{}^2)^{-1} = \begin{cases} -\dfrac{1}{|q_s|}\arctan[(2|q_s|)^{-1}] & q_s{}^2 < 0 \\ \dfrac{1}{2q_s}\log[(1 - 2q_s)/(1 + 2q_s)] & q_s{}^2 > 0 \end{cases} \tag{3.25}$$

Since the imaginary part of $\Lambda$ is positive, the pole at the $\rho$ mass is on a higher sheet.

In Table 2, I have listed some selected values of the phase shift as predicted by the TP model with only the $\pi$-$\pi$ channel included, and those predicted by this approximate renormalized amplitude. As can be seen, there is virtually no difference between the predictions of the two models. As with the $\pi$-$\pi$ predictions of the TP model, the best results are obtained from the renormalization approach with $\Gamma_\rho \simeq 150$ MeV.

I have also investigated the negative $s$ region of the renormalized amplitude, and find the denominator of equation (3.22) to be negative definite. Thus, the renormalized amplitude has no ghost poles. Note also, that in the renormalized approach, there are only two input parameters, the $\rho$ mass and width. Also, in this approach, the $\pi$-$\pi$ channel is essentially exclusively responsible for the creation of the $\rho$.

Table 2. Comparison of representative $\pi\pi$ phase shift predictions of TP model with the $\pi$-$\pi$ channel only, renormalization approach, and experimental data. All angles are in degrees.

| $s/\mu^2$ | TP predictions | Renormalization approach predictions | Experimental data |
|---|---|---|---|
| 12.8 | 6.0 | 6.4 | 8.1 ± 5.2 |
| 18.2 | 14.8 | 15.6 | 17.4 ± 3.4 |
| 24.9 | 42.9 | 44.0 | 42.2 ± 5.0 |
| 32.6 | 111.1 | 110.6 | 109.6 ± 8.4 |
| 41.1 | 141.8 | 141.4 | 141.4 ± 4.1 |
| 50.6 | 151.6 | 151.6 | 148.7 ± 4.1 |
| 61.1 | 156.0 | 156.4 | 151.8 ± 5.6 |
| 72.9 | 158.4 | 159.2 | 166.9 ± 7.6 |

## 4. *Form Factors and Scattering Lengths*

A comparison of the pion form factor predictions of the two models, yield essentially identical results, as they must. Both show reasonable agreement with experiment (Augustin *et al.*, 1968; Auslander *et al.*, 1968; Gounaris, 1969; Parkinson, 1970).

Writing $T_{\pi\pi}(s) \sim D^{-1}(s)$, the form factor is

$$F_\pi(s) = D(0)/D(s) \tag{4.1}$$

These results are displayed by Parkinson (Parkinson, 1970). The best fits to the form factor data in both models occur for $m_\rho = 770\,\text{MeV}$, $\Gamma_\rho = 109\,\text{MeV}$. This is in contrast to the 150 MeV width needed for good phase shift predictions (see footnote 1).

Tryon has shown (Tryon, 1971) that the $p$-wave scattering length can be accurately related to the $p$-wave phase shift by

$$\delta_1{}^1 = a_1(s - 4\mu^2)^{3/2}(16s)^{-1/2} \tag{4.2}$$

The values of $a_1$ predicted by the two models, along with experimental results (which were deduced by Tryon from $Ke_4$ data) are given in Table 3.

[1] In performing the computations of the form factor and scattering length from the TP model, $C_1{}^2$ was adjusted for the various $\rho$ widths quoted, by setting

$$\operatorname{Im} D(s)\,|_{s=m^2} = -m_\rho \Gamma_\rho$$

$D(s)$ being given in equation (2.1).

Table 3. Comparison of TP and renormalization model predictions of $p$-wave scattering length $(a_1\mu^2)$ to experimental data.

| TP predictions $\Gamma_\rho \simeq 140\,\mathrm{MeV}$ | Renormalization model predictions $\Gamma_\rho \simeq 140\,\mathrm{MeV}$ | Experimental data (Tryon, 1971) |
|---|---|---|
| .039 | .040 | .037 ± .003 (Beier *et al.*, 1973) |
| | | .042 ± .006 (Zylbersztejn *et al.*, 1972) |
| | | $.037\,^{+\,.007}_{-\,.010}$ (Schweinberger *et al.*, 1971) |
| | | .045 ± .005 (Ely *et al.*, 1969) |

## 5. *Conclusions*

I have considered two models of the $\rho$ meson and the approximate low energy $\pi$-$\pi$ amplitude which can be obtained from them. Both approaches make essentially the same predictions of various scattering parameters which are in reasonable agreement with experimental data. The Tschang–Parkinson model, which is parametrized by a fit to $\pi$-$\pi$ phase shift data, seems to contain ghosts. It also contains parameters which are adjusted by fitting the phase shift data. The renormalized amplitude requires only the $\rho$ mass and width to completely specify it, and it does not have the negative energy pole which seems to be present in the TP model.

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